A SYSTEMATIC APPROACH TO STATE FEEDBACK CONTROLLER DESIGN FOR DC/DC LINE-SIDE TRACTION CONVERTERS

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Abstract. The paper presents a systematic approach to state feedback controller design for DC/DC line-side traction converters. The general approach is demonstrated on one particular converter, which can be regarded as a multi-input multi-output system. It results in good closed-loop system dynamics even when no resistances are present.

Keywords. Multi-input multi-output systems, DC/DC converter, line-side converter, traction converter, state feedback

## INTRODUCTION

For modern traction drives with DC links at DC power supplies DC/DC line-side converters, also known as choppers, are needed to set-up or set-down the voltage. A simple chopper is usually controlled by a cascaded structure of PI controllers. However, application of those simple choppers for locomotives operating at high DC voltage power supplies is not yet possible due to the blocking voltage and maximum current rating of the GTOs or IGBTs available to date. For that reason series or parallel connections of chopper elements are required for high power converters at higher supply voltages, c. f. for example Geißler and Unger-Weber [1]. One particular practical application of those choppers in a traction drive is mentioned in Weschta [2]. Some of those DC/DC converters are depicted in figures 1 to 4.

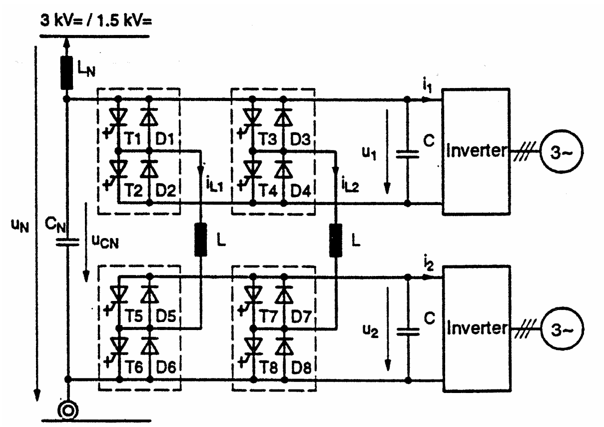


Figure 1: Buck/boost converter

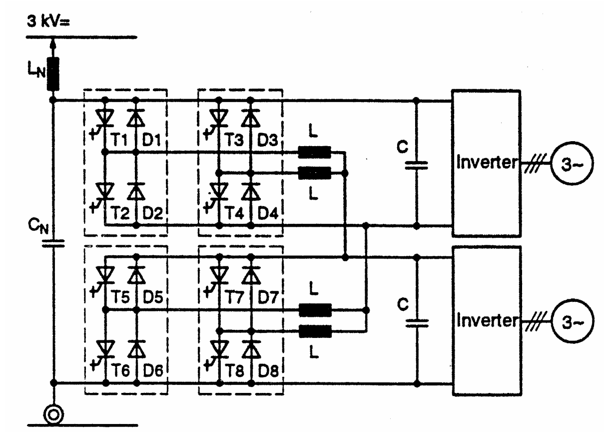


Figure 3: Buck converter

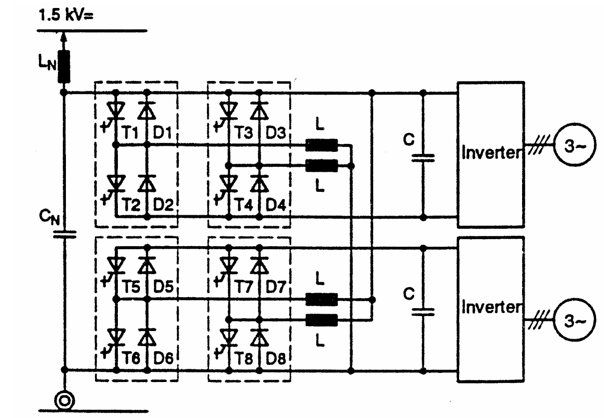


Figure 2: Boost converter

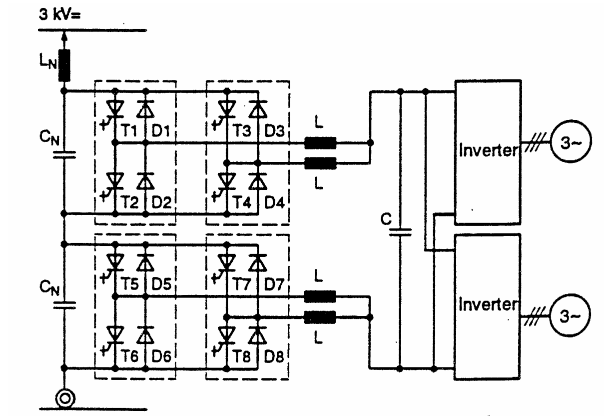


Figure 4: Buck converter

From the control point of view those choppers are multi-input multi-output (MIMO) systems: For example, in figures 1 through 3 two output voltages are to be kept at specified values independent of load conditions, using four gating signals. At the same time currents in chokes arranged in parallel should be equal - at least under symmetrical load conditions - in order to ensure identical loads of chokes and phase modules. It is seen that ideally no resistors are present so that in reality the circuits are weakly damped systems.

This paper addresses control of those above-mentioned choppers. Looking at one particular chopper, one might think of using some form of decoupling plus a number of PI controllers to achieve the control objective. General strategies for designing decoupling controllers for MIMO-systems are described in Tolle [3]. A special decoupling strategy for one particular of the choppers pictured below is given in Tiedtke [4] However, it is not obvious how such decoupling controllers for weakly damped systems should be generally structured and parametrized to give the controlled system the required closed-loop dynamics. Furthermore, it is not even obvious that such an approach will work for all DC/DC converters in figures. 1 through 4 and the many more that may exist.

Hence in this paper we propose to use state feedback controllers (see e. g. Kailath [5]) since these controllers can be systematically designed for a variety of DC/DC converters. Furthermore, state feedback is well suited for controlling weakly damped systems. Algorithms for state feedback and related observer design are proven and are available in many commercial software packages. State feedback for DC/DC converters has been described in the literature before, c. f. Leung and Tam [6]. However, applications were limited to single output systems such as a Cuk converter circuit.

Controllers for traction converters are usually implemented on digital hardware. Specific features of discrete-time control of pulse-width modulated systems such as computational delay between controller inputs and outputs can be naturally incorporated in the design: Using the state-space model obtained during design, a predictor can be obtained to compensate for that computational delay. Also it is possible to include a state observer for a selected observable sub-system to dispense with some measured variables (e. g. for reason of cost to measure those variables).

From the state-space modelling - as a sort of by-product - important information on certain system properties, such as controllability of certain states as a function of operating conditions and design parameters, can be gathered.

## CONTROL SYSTEM STRUCTURE

The general control system proposed for DC/DC line-side converters is as depicted in figure 5. The DC/DC converter to be controlled has a number of pulse-width modulated (PWM) gating signals $\alpha_i$ as inputs, depending on the number of phase moduls in the power circuit. System outputs are usually two DC link voltages $u_1$ and $u_2$ or one DC link voltage for the special converter depicted in figure 4. From the control point of view changes in input voltage (line voltage $u_N$) and load currents $i_1$ and $i_2$ (or the single load current for the converter in fugure 4) are interpreted as disturbances of the control system. Note that notation in figure 5 is adapted to the special case of the converter according to figure 1.

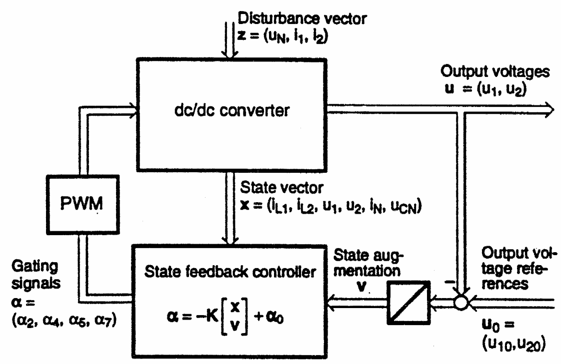


Figure 5: State feedback control structure

The main task of the control system is to keep output voltages at fixed values $u_{10}$ and $u_{20}$ regardless of line voltage and load currents. To ensure zero steady-state error the system state is augmented by integrators for each DC link voltage to be controlled. The state feedback controller consists basically of a (m×n) matrix K of numbers (m is the number of system inputs, n is the system order) to be multiplied with the system state vector. The steady state solution $\alpha_0$ for the system input vector is added for fast controller response. Additional features of the proposed state feedback controller are, for example, easy discrete-time implementation, including prediction, and possible use of observed state variables instead of measured ones. Also, it is advantageous to synchronize controller operation and pulse-width modulation.

## DESIGN PROCEDURE

As can be seen from figures 1 through 4, there is a variety of power circuits for high power DC/DC traction converters. It is obvious that those choppers may behave rather differently from the control point of view. However, the advantage of the proposed state feedback approach to controller design is that it can be applied in more or less the same fashion to all those circuits. The result is a generic design procedure that can be described in more detail as follows.

**Structure**

Obtain the basic power-electronic circuit for the particular DC/DC converter including parameters for chokes and capacitors. Operating conditions such as possible input voltage, desired DC link voltage and load conditions are also necessary.

**Modelling**

Set up the continuous-time system model: The gating signals for the GTOs or IGBTs will be pulse-width modulated (PWM). Assuming that the switching frequency is high as compared to the system eigenfrequencies, the method of state-space averaging can be applied. Hence the continuous-time state space model can be set up from the various system differential equations. It can be augmented by integrators to achieve zero steady-state error of the system outputs if necessary. Also, it turns out to be useful to incorporate the line filter into the system model. Due to its comparatively low eigenfrequencies the line filter is the converter element most limiting to the overall system dynamics.The state-space model will generally be a nonlinear one with duty cycles of the gating signals entering multiplicatively, i. e. as elements of the system matrix.

Determine the systems steady-state and linearize the state-space model around a general steady-state solution. Here system properties such as local controllabilty or observability can be investigated.

Compute the discrete-time state space model for various selected operating points by discretization using the sampling frequency. Usually the sampling time will be an integer multiple of the pulse time.

**State feedback design**

Compute a state feedback controller via pole placement or LQR design. The latter approach is advantageous, since the system dynamics can be adjusted by means of only a few design parameters, c. f. e. g. Kwakernaak and Sivan [7]. Here algorithms of commercial control engineering software packages are readily applicable.

If necessary, determine an observer (full-order or reduced-order Luenberger observer) for an observable subsystem in order to reduce the number of variables to be measured. Here also LQ design or a suitable pole placement strategy can be applied.

Control system performance can be improved by incorparating a predictor for system variables to cope with computational delays within the control system and with delays caused by the PWM converter operation. Also, since for traction application system operating conditions may vary in a wide range, there should be different controller designs for those different operating points.

**Simulation and testing**

Simulation of the DC/DC converter, including control, permits first insight into closed-loop system behavior. Controller design can possibly be improved as a result of simulations by modifying, e. g., design parameters for LQR state feedback controller design.

After implementation on digital hardware the system should be tested using an analog or quasi-analog simulation of the power electronic circuit. Control commissioning with the real power electronic circuit is the final step.

**EXAMPLE**

In what follows the above design algorithm will be applied to design a state feedback controller for the buck/boost converter depicted in figure 1. Control inputs of the converter are PWM gating signals for each phase modul. The converter will be operated in such a way that all four phase moduls are pulsing all the time. This makes sure the converter will operate continuously when the line voltage is close to the desired DC link voltage. Here semiconductors indexed 2, 4, 5 and 7 are used for control, gating signals for indices 1, 3, 6 and 8 are the respective inverses. Interpreting, for the time being, $\alpha_2$, $\alpha_4$, $\alpha_5$ and $\alpha_7$ as continuous-time signals in the range between 0 and 1, the circuit can be modelled by the following set of differential equations:

$$
\begin{aligned}
L \frac{d}{dt} i_{L1} &= -\alpha_2\, u_1 - \alpha_5\, u_2 + u_{CN} \\
L \frac{d}{dt} i_{L2} &= -\alpha_4\, u_1 - \alpha_7\, u_2 + u_{CN} \\
C \frac{d}{dt} u_1 &= \alpha_2\, i_{L1} + \alpha_4\, i_{L2} - i_1 \\
C \frac{d}{dt} u_2 &= \alpha_5\, i_{L1} + \alpha_7\, i_{L2} - i_2 \\
L_N \frac{d}{dt} i_N &= -u_{CN} + u_N \\
C_N \frac{d}{dt} u_{CN} &= -i_{L1} - i_{L2} + i_N
\end{aligned}
\tag{1}
$$

Steady-state solution of this system is

$$
\begin{aligned}
\begin{bmatrix} i_{L10} \\ i_{L20} \end{bmatrix} &= \begin{bmatrix} \alpha_2 & \alpha_4 \\ \alpha_5 & \alpha_7 \end{bmatrix}^{-1} \begin{bmatrix} i_{10} \\ i_{20} \end{bmatrix} \\
\begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} &= \begin{bmatrix} \alpha_2 & \alpha_5 \\ \alpha_4 & \alpha_7 \end{bmatrix}^{-1} \begin{bmatrix} u_{CN0} \\ u_{CN0} \end{bmatrix} \\
\begin{bmatrix} u_{CN0} \\ i_{N0} \end{bmatrix} &= \begin{bmatrix} u_{N0} \\ i_{L10} + i_{L20} \end{bmatrix}
\end{aligned}
\tag{2}
$$

Clearly, when both DC link voltages are to be kept at identical values (say $u_{10} = u_{20} = u_0$), and when for reason of symmetry two gating signals are chosen equal in steady-state operation ($\alpha_{20} = \alpha_{70}$, $\alpha_{40} = \alpha_{50}$), then

$$u_{N0} = (\alpha_{20} + \alpha_{40})\, u_0$$
$$\begin{bmatrix} i_{L10} \\ i_{L20} \end{bmatrix} = \begin{bmatrix} \alpha_{20} & \alpha_{40} \\ \alpha_{40} & \alpha_{20} \end{bmatrix}^{-1} \begin{bmatrix} i_{10} \\ i_{20} \end{bmatrix} \tag{3}$$

Hence the sum $\alpha_{20}+\alpha_{40}$ is defined by DC link and line voltages, whereas individual values for $\alpha_{20}$ and $\alpha_{40}$ must be determined from load currents and currents in converter chokes. Clearly, under realistic conditions load currents $i_1$ and $i_2$ will never be identical. It is seen from equation (3) that the matrix consisting of the gating signals has to be nonsingular in order to ensure finite currents $i_{L1}$ and $i_{L2}$ in both chokes under those realistic load conditions. Hence the straight-forward steady state solution of the system with $\alpha_{20} = \alpha_{40} = \alpha_{50} = \alpha_{70}$ is not feasable. Instead steady state gating signals have to be chosen in an asymmetrical way, for example as

$$\begin{aligned} \alpha_{20} &= \alpha_{70} \\ &= \gamma\,(\alpha_{20} + \alpha_{40}) \\ &= \gamma\, \frac{u_{N0}}{u_0} \\ \alpha_{40} &= \alpha_{50} \\ &= (1-\gamma)\,(\alpha_{20} + \alpha_{40}) \\ &= (1-\gamma)\, \frac{u_{N0}}{u_0} \end{aligned} \tag{4}$$

for some $\gamma$ in the range between 0.5 and 1. Chosing $\gamma$ close to 0.5 means that various phase moduls will have near equal load, which is clearly desirable in terms of semiconductor losses. However, in case of asymmetrical load currents this design will result in high currents through converter chokes. Chosing $\gamma$ close to unity means good system behavior in terms of currents in chokes when loads are asymmetrical but will result in an uneven distrubution of losses amoung phase modules. As a compromise, in what follows $\gamma$ was chosen as $\gamma = 2/3$.

Next the linearized state space model is computed as

$$\frac{d}{dt} x = A\, x + B_u\, u + B_z\, z \tag{5}$$

where

$$x = \begin{bmatrix} i_{L1} - i_{L10} \\ i_{L2} - i_{L20} \\ u_1 - u_{10} \\ u_2 - u_{20} \\ i_N - i_{N0} \\ u_{CN} - u_{CN0} \end{bmatrix} \quad u = \begin{bmatrix} \alpha_2 - \alpha_{20} \\ \alpha_4 - \alpha_{40} \\ \alpha_5 - \alpha_{50} \\ \alpha_7 - \alpha_{70} \end{bmatrix} \quad z = \begin{bmatrix} u_N - u_{N0} \\ i_1 - i_{10} \\ i_2 - i_{20} \end{bmatrix} \tag{6}$$

and the system matricies are defined as follows:

$$A = \begin{bmatrix} 0 & 0 & \frac{-\alpha_{20}}{L} & \frac{-\alpha_{50}}{L} & 0 & \frac{1}{L} \\ 0 & 0 & \frac{-\alpha_{40}}{L} & \frac{-\alpha_{70}}{L} & 0 & \frac{1}{L} \\ \frac{\alpha_{20}}{C} & \frac{\alpha_{40}}{C} & 0 & 0 & 0 & 0 \\ \frac{\alpha_{50}}{C} & \frac{\alpha_{70}}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_N} \\ \frac{-1}{C_N} & \frac{-1}{C_N} & 0 & 0 & \frac{1}{C_N} & 0 \end{bmatrix}$$

$$B_u = \begin{bmatrix} \frac{-u_{10}}{L} & 0 & \frac{-u_{20}}{L} & 0 \\ 0 & \frac{-u_{10}}{L} & 0 & \frac{-u_{20}}{L} \\ \frac{i_{L10}}{C} & \frac{i_{L20}}{C} & 0 & 0 \\ 0 & 0 & \frac{i_{L10}}{C} & \frac{i_{L20}}{C} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{-1}{C} & 0 \\ 0 & 0 & \frac{-1}{C} \\ \frac{1}{L_N} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

Two integrators for the DC link voltages are easily added to the linearized state-space model to ensure zero steady state error of the output voltages. The buck/boost converter according to figure 1 is to be operated at a pulse frequency of 250 Hz. Hence the sampling time for discretization of the linearized system model is chosen as T = 2 ms, i. e. sampling and control is done twice in one pulse period. For the resulting discrete-time system model a state feedback controller is designed using the LQR approach with suitable weighting matricies [7]. Both above steps can be easily performed by commercially available control engineering software. Note that the line filter is included in the model used for controller design.

The resulting controller was implemented on digital hardware including a predictor to compensate for the computational delay. Figures 6 through 8 show some results for step-like load changes (0 MW to 1 MW per DC link) and for variations of desired DC link voltages (2100 V to 2400 V) and line voltage (1500 V to 1800 V) showing that the system can be controlled in a dynamically acceptable way.

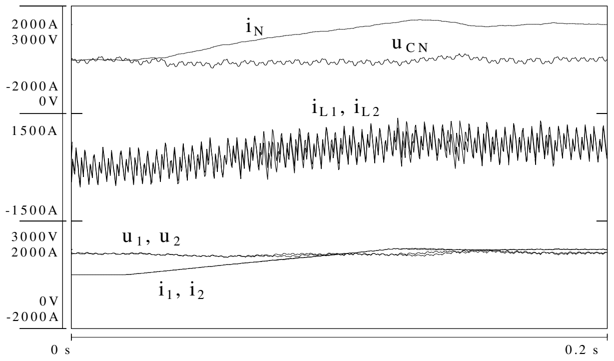


Figure 6: System response to load changes of 0 MW to 1 MW per DC link

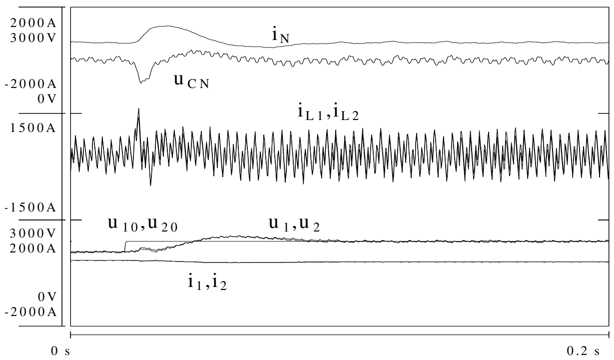


Figure 7: System response to step-like changes of desired DC link voltages from 2100 V to 2400 V

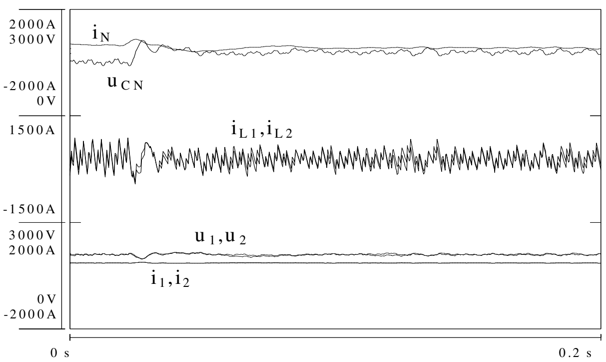


Figure 8: System response to step-like change of line voltage from 1500 V to 1800 V

## CONCLUSIONS

The paper proposes to use state feedback for control of DC/DC converters. Here special attention is given to those choppers that are - from the control point of view - multi-input multi-output systems. The paper presents a systematic approach to design of state feedback controllers for a variety of such DC/DC converters. The advantage of the proposed approach is that it can be applied more or less in the same fashion to a number of DC/DC converters. From a variety of choppers given as possible applications, the approach has been demonstrated on one particular buck/boost converter. However, the proposed control method is applicable to a variety of other DC/DC converters. The approach yields good closed-loop system dynamics even if no damping by resistances is present in the system.

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