Coalition formation versus free riding in rent-seeking contests*

Lukas Block

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Abstract

We study lobby group formation in a two-stage model where the players first form lobby groups that then engage in a rent-seeking contest to influence the legislator. However, the outcome of the contest affects all players according to the ideological distance between the implemented policy and the players' preferences. The players can either lobby by themselves, form a coalition of lobbyists or free ride. We find that free coalition formation is reasonable if either players with moderate preferences face lobby groups with extreme preferences, or if there are two opposing coalitions with an equal number of members. Otherwise, there are always free riders among the players.

Keywords: Group formation, Rent-seeking, Free riding

JEL Classification Numbers: C71, D72, D74

*Department of Economics, University of Paderborn, Warburger Str. 100, 33098 Paderborn, Germany.

1 Introduction

In today's democracies the legislation process is highly complex. It takes both time and effort to transform a first idea into a law that can be passed by the parliament. Furthermore, a first draft law is usually altered various times before the ballot. The major reason for the complexity of the process is the high number of groups or players that are involved. Depending on the parliamentary system, these players include parties, the members of parliament, the government and the ministries on the inside of the legislation process. Additionally, there are lobby groups on the outside of that process aiming to lead the results towards their favorable status.

These lobby groups and their activities have been in the center of research interest. In the field of rent-seeking literature, both political scientists and economists study the way how firms or individuals try to influence the legislation process. One key aspect is the formation of lobby group, or the question under which conditions are individuals willing to cooperate. From the rent seekers' perspective, lobbying follows the logic of a public good problem: It may be beneficial for a player that lobbying takes place. However, participation includes some costs such that there is the incentive to free ride on other players' expenses.

This paper addresses the question at which turning point rent-seekers decide to become an active lobbyist instead of free riding on the lobbying activities of others. Lobby groups engage in a rent-seeking contest, but the outcome does not only affect the winning group. Instead, the prize of winning the contest is choosing the policy that is implemented by the legislator. However, that policy is valid for all members whether they are in favor of it or not. So far the literature has paid little attention on the spillover effects that are generated when rent-seekers evaluate their prospects of the contests.

For that matter we build a model that consists of four players that can choose to form a lobby group by themselves, do coalition formation and join a lobby group with other players, or abstain from the lobbying process and free ride. Afterward, all lobby groups engage in the rent-seeking contest. The players have cardinal preferences over a one-dimensional policy such that they can quantify an ideological distance between each potential policy and their own preference. Also we assume linear lobbying costs such that they can compute the costs and potential benefits of each lobby group structure. Therefore, the rather small set of players allows us to identify the border in a basic setup, when players have the incentive to work together with one other player instead of just being the free rider of that player.[1]

Our major result is the identification of the stable lobby group structures where neither the free riders would prefer to become lobbyists nor the lobbyists prefer to become free riders. Concretely,

[1] An interesting connection to the real life with the huge amount of all different kinds of lobby groups could be the following: If there are lobby groups nominally close interests, at which point can we expect collaboration? Also, if there are groups whose member have rather different interests, at which point does the group split up or do members drop out?

we quantify the distance in terms of political preferences between the lobby groups. Coalition formation is only possible in two scenarios: Either there are three lobby groups and the group in the middle with rather moderate preferences is a coalition, or there are two lobby groups that both contain two players. In all other lobby group constellation there is at least one free rider, who is more likely to have rather moderate preferences.

2 Related literature

The seminal work of Olson (1965) initiated a large literature on collective action that also explores the connection between the group size and the group effort. He identified the "paradox of group size" and argued that it is easier for smaller groups to work efficiently and control the free riding. This model also follows this logic even though it only contains four players. Corchón (2007) extends the analysis that group size rather depend different valuations or costs structures of the groups.

There are more factors which induce coalition formation.[2] Becker (1983) introduces homogeneity as an indicator for successful lobby groups. He argues that it easier for groups to control for free riding, the more homogenous they are, which is quantified in this model. In this context Anesi (2009) distinguishes between the free riding in between groups and within groups. Whereas Pecorino (1998) analyzes how cooperative behavior within a lobby group can be maintained in a repeated tariff lobbying game. In our model there is no way to prevent free riding, such that players will always free ride if it is beneficial for them.

Another plausible explanation for the outcome of coalition formation from Nti (2004) is an asymmetric valuation for the legislative outcome of the involved players, which can interpreted as an ideological factor. Further Bloch (2012) points out that the size of groups is related to the nature of the prize that can contain both elements of a private and public good. The contribution of **?** is close to our work as it considers coalition formation in the light of contests.[3]

An important role is also allocated to the legislator. The relation between the lobby groups favorable positions and the legislator's inherent one may determine the positioning of a lobby group. Motivational reasons both for supporting the legislator (Hojnacki and Kimball (1998)) and for opposing the legislator (Felli and Merlo (2006)) can be found. Additionally an active role of the legislator is discussed. In that case the legislator is fulfills the role as a mechanism designer, in order to induce a socially optimal result. Thus, forming a lobby group can also be a result of the designed political environment as in Amegashie (1999), Michaels (1988) or Dasgupta and Nti (1998).

The results of coalition formation depends on the process of formation as well. In this model we use a framework of Mitra (1999) who first incorporated coalition formation as a separate step in within collective actions. Then there exists the possibility for players to write contracts before

[2]Pioneering work from Gamson (1961) provides profound characteristics of coalition formation games.

[3]However, he focuses on rivalry among group members which we do not.

joining together. Examples are Ray and Vohra (1997) who study refinement of coalition structures, Hyndman and Ray (2007) who considers history-dependence, or Yi (1997) who compares different contracts. In addition, Konishi and Ray (2003) characterizes coalition formation as a dynamic process.

In this paper we use a version of Nash stability to describe our lobby group structure. More precisely, we consider a structure to be stable if no player unilaterally deviates and no two players wish to form a group either. Other stability concepts have also enriched the discussion. Hart and Kurz (1983) study the formation of players according to the division rule based on the Shapley value, where deviation of a player either leads to breakdown of the entire coalition structure or just the affected coalition. Further Chwe (1994) takes farsightedness of into account.

3 The Model

In order to describe lobbying in a collective-action setting, we use a two-stage model that was introduced by Mitra (1999). In the first step, there is the lobby group formation. Individuals have the choice to either form a lobby group by themselves, join a lobby group with other individuals, or abstain from any lobbying activity. In the second step the rent-seeking contest takes place with all groups that have been formed.

Let there be a set of players, N, that contains four players. Each player has preferences over a one-dimensional policy ρ that depicts the political sphere from -1 to 1. Denote with ρ_i player i's most preferred policy and let $\rho_1 \leq \rho_2 \leq \rho_3 \leq \rho_4$. If all preferences are different we can call players 1 and 4 corner players which have rather extreme preferences, and players 2 and 3 center players which have rather moderate preferences. We assume that the preferences over policies are cardinal, i.e. the players can calculate the distance between their political positions which is common knowledge among them.

A lobby group $S_k \subset N$ is a group of players that contains at least one player. Let there be at most m lobby groups, which are counted in roman numbers such that S_k is the k-th lobby group with $k \in \{I, .., m\}$ and $m \leq 4$ if we assume that a player does not join more than one group. Each lobby group also favors a certain policy denoted with ρ_k. Lobby groups with only one player naturally represent her interest, while in groups with many players they have to agree on one policy. Once the lobby group have decided on a prefered policy, we also order them with $\rho_k \leq \rho_{k+1}$. Further, denote with σ the set of lobby groups and with z_k the amount of players in S_k.

The policy that is implemented is determined via a rent-seeking contest, which is based on Tullock (1980). This contest follows the logic of a lottery in which lobby groups can buy lottery tickets. There is a legislator who is ex-ante indifferent and randomly draws one lottery ticket out of the box that contains all bought tickets.[4] The difference to model is that lobby groups instead

[4]In our model, the legislator is agenda-neutral and does not consider social welfare.

of individuals make the decision how many tickets should be bought. Hence, the chance of a lobby group to win the contest and thus persuade the legislator is equal to its share of the total lobby expenses. If a lobby group only consists of one player, she has to cover the entire expenses, while for groups with many players the costs are equally shared among the group members. Let r_i be the expenses of player i and R_k be the expenses of lobby group S_k. R_k is strictly positive and not limited, since it represents the decision of the lobby group how much to invest in lobbying. In contrast, r_i may be zero if player i is a free rider.

We can also interpret r_i as a membership fee for player i of being part in S_k, such that $r_i = \frac{1}{z_k} R_k$. We denote with $\mathcal{R}(\sigma)$ the vector of all lobby group expenses and with $R = \sum_{k=I}^{m} R_k$ the sum of expenses. Then we can formalize the winning probability p of convincing the legislator as a function of expenses with the use of a contest success function:[5]

$$p_k(\mathcal{R}) = \frac{R_k}{R}$$

Lobby group S_k wins the rent-seeking contest and the legislator passes a law that corresponds to ρ_k. As that law accounts for all player, the outcome of the contests affects each player, whether or not she belongs to the winning coalition. We normalize the utility gain for player i to 1 if $\rho_i = \rho_k$, and 0 if $|\rho_i - \rho_k| = 1$. As the political sphere has a length of 2, players can also receive negative utility if a policy of the opposing side is implemented. In addition, we assume that a player's utility is single-peaked and decreases linearly. Hence, the expected utility function for player i is

$$u_i(\sigma, \mathcal{R}) = \big[\sum_{k=I}^{m} (1 - d_{ik}) p_k(\mathcal{R})\big] - r_i,$$

while $d_{ik} = |\rho_i - \rho_k|$ denotes the distance between the player i and lobby group k.

The same logic appears when we now look at the expected profit functions of the lobby groups. Let the lobby group be represented by an artificial representative. This representative adopts the preference ρ_k that the members of the coalitions agreed upon. Again, the lobby group gains some value if it wins or loses the contest, depending on the distance towards the winning lobby group. In addition, the representative considers the number of members of the lobby groups when choosing R_k. As the expenses of the lobby groups are common knowledge we can compute the expected profit function π of a fixed lobby group l. We get:

$$\pi_l(\sigma, \mathcal{R}) = \big[\sum_{k=I}^{m} (1 - |\rho_l - \rho_k|) p_l(\mathcal{R})\big] - \frac{R_k}{z_k}.$$

To facilitate further reading, we denote the distance between lobby group I and II with α, between II and III with β, between III and IV with γ, and last $\delta = \alpha + \beta$.

Note that profit function π_k and and the utility u_i function coincide whenever a player is the only member of a lobby group. However it could both be the case that either the lobby group of a player does not represent her exact preferences or that she does not have any cost if she is a free rider.

[5] See Skaperdas (1996) for an axiomatization of the contest success function.

The strategy s_i of players i is the decision to either free ride or to join a lobby group. Let free riding be equivalent to joining lobby group 0, so formally $s_i \in \{0, I, ..., m\}$. Denote with s the vector containing all players' strategies. Hence, the lobby group structure is a result of the player' strategies.

We can now formulate the stability concept of the model. A lobby group structure $\sigma(s)$ is stable, if the following holds:

1. No player wants to leave a coalition to become a free rider,
2. no player unilaterally wants to form a coalition, and
3. no two players want to bilaterally form a coalition.

Formally, we have

$$\begin{aligned} u_i(\sigma(s), \mathcal{R}) &\geq u_i(\sigma(s_i', s_{-i}), \mathcal{R}), \\ u_i(\sigma(s), \mathcal{R}) &\geq u_i(\sigma(s_i', s_j', s_{-ij}), \mathcal{R}) \text{ and} \\ u_j(\sigma(s), \mathcal{R}) &\geq u_j(\sigma(s_i', s_j', s_{-ij}), \mathcal{R}), \end{aligned}$$

where s_i' denotes deviating player i and s_{-i} the strategy vector without her.

4 Results

As the players have the options to either engage or abstain from the rent-seeking contest, there are different coalition structures possible. We will now analyze the different constellations while first computing the optimal investments by the lobby groups and then the incentives of the players to be part of these groups.

i) *Four lobby groups*

In this constellation, there are four active lobby groups, each group has only one member and there is no free riding. The expected profit functions of lobby group k reads:

$$\begin{aligned} \pi_k(\sigma, \mathcal{R}) = & \frac{(1 - |\rho_k - \rho_I|)R_I + (1 - |\rho_k - \rho_{II}|)R_{II}}{R_I + R_{II} + R_{III} + R_{IV}} \\ & + \frac{(1 - |\rho_k - \rho_{III}|)R_{III} + (1 - |\rho_k - \rho_{IV}|)R_{IV}}{R_I + R_{II} + R_{III} + R_{IV}} - R_k \end{aligned} \tag{1}$$

Before we take a close look we also consider the second constellation.

ii) *Three lobby groups with one member each:*

Here we find three lobby groups that contain one member each. Hence, there is one free rider among the players. For each lobby groups we get this expected profit function:

$$\begin{aligned} \pi_k(\sigma, \mathcal{R}) = & \frac{(1 - |\rho_k - \rho_I|)R_I + (1 - |\rho_k - \rho_{II}|)R_{II}}{R_I + R_{II} + R_{III}} \\ & + \frac{(1 - |\rho_k - \rho_{III}|)R_{III}}{R_I + R_{II} + R_{III}} - R_k \end{aligned} \tag{2}$$

Given these setup we can directly gather the first result:

Proposition 1. *There is no stable lobby group structure σ that contains three or four lobby groups, which have one member each.*

Proof. The first order conditions of (1) for players 1 to 4 in are:

$$\alpha R_{II} + (\alpha+\beta)R_{III} + (\alpha+\beta+\gamma)R_{IV} = (R_I + ... + R_{IV})^2 \tag{3}$$

$$\alpha R_I + \beta R_{III} + (\beta+\gamma)R_{IV} = (R_I + ... + R_{IV})^2 \tag{4}$$

$$(\alpha+\beta)R_I + \beta R_{II} + \gamma R_{IV} = (R_I + ... + R_{IV})^2 \tag{5}$$

$$(\alpha+\beta+\gamma)R_I + (\beta+\gamma)R_{II} + \gamma R_{III} = (R_I + ... + R_{IV})^2 \tag{6}$$

Solving this system of equations yields:

$$(3)\&(4) \Rightarrow R_I = R_{II} + R_{III} + R_{IV}$$

$$(4)\&(5) \Rightarrow R_I + R_{II} = R_{III} + R_{IV}$$

$$(5)\&(6) \Rightarrow R_{IV} = R_I + R_{II} + R_{III}$$

This can only be solved if $R_{II} = R_{III} = 0$, such that players 2 and 3 would be better off with free riding. Therefore, four lobby groups cannot be active.

The same argumentation holds true for case *ii*). The FOCs of (2) for the three lobby groups are:

$$\alpha R_{II} + \delta R_{III} = (R_I + R_{II} + R_{III})^2$$

$$\alpha R_I + \beta R_{III} = (R_I + R_{II} + R_{III})^2$$

$$\delta R_I + \beta R_{II} = (R_I + R_{II} + R_{III})^2$$

Again, we can only solve it with $R_{II} = 0$. Hence, there cannot be three active lobby groups with only one member each. □

For the interpretation we can distinguish between players 1 and 4 who have relatively extreme preferences and players 2 and 3 who have relatively moderate preferences. If both extreme players are active, then their preference are also adopted by the lobby group which they are a part of. The distance between players 1 and 4 is the largest among the players, therefore they are willing to spend more resources. In contrast the relatively moderate players 2 and 3 do not see the same pressure to become active and thus prefer to free ride.

iii) *Three lobby groups and one is group is a coalition:*

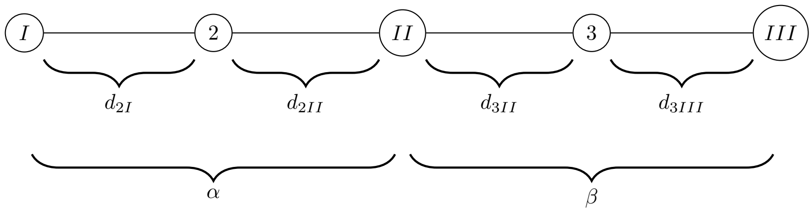

Figure 1: Three lobby groups and S_{II} is a coalition

In this constellation all players are active and players 2 and 3 form coalition S_{II}. The expected profit function of the lobby groups are:

$$
\begin{aligned}
\pi_I(\sigma,\mathcal{R}) &= \frac{R_I + (1-\alpha)R_{II} + (1-\delta)R_{III}}{R_I + R_{II} + R_{III}} - R_I \\
\pi_{II}(\sigma,\mathcal{R}) &= \frac{(1-\alpha)R_I + R_{II} + (1-\beta)R_{III}}{R_I + R_{II} + R_{III}} - 1/2R_{II} \\
\pi_{III}(\sigma,\mathcal{R}) &= \frac{(1-\delta)R_I + (1-\beta)R_{II} + R_{III}}{R_I + R_{II} + R_{III}} - R_{III}
\end{aligned}
$$

In contrast to the previous case where players 2 and 3 were by themselves, now they collaborate and share the costs. This gives the following result.

Proposition 2. *A lobby group structure $\sigma = \{S_I, S_{II}, S_{III}\}$ and $z_{II} = 2$ is stable iff*

(a) $3\alpha \geq \beta \geq \frac{\alpha}{3}$,

(b) $d_{2II} \leq \frac{1/4\delta^3 - 2\alpha\beta\delta}{\alpha^2-\beta^2-\delta^2}$ *and*

(c) $d_{3II} \leq \frac{1/4\delta^3 - 2\alpha\beta\delta}{\beta^2-\alpha^2-\delta^2}$

Proof. First we compute the FOCs of each player.

$$\alpha R_{II} + \delta R_{III} = (R_I + R_{II} + R_{III})^2 \tag{7}$$

$$2\alpha R_I + 2\beta R_{III} = (R_I + R_{II} + R_{III})^2 \tag{8}$$

$$\delta R_I + \beta R_{II} = (R_I + R_{II} + R_{III})^2 \tag{9}$$

To solve this system of equations we calculate with the left hand side of

$$
\begin{aligned}
(7)\&(8) \text{ and } (7)\&(9) \Rightarrow \quad & R_I = R_{II}\left(\tfrac{\beta(3\alpha-\beta)}{\delta^2}\right) \\
(7)\&(8) \text{ and } (8)\&(9) \Rightarrow \quad & R_{III} = R_{II}\left(\tfrac{\alpha(3\beta-\alpha)}{\delta^2}\right).
\end{aligned}
$$

Inserting this back to equation (8) gives us the optimal investment of the lobby groups with

$$
\begin{aligned}
R_I &= \tfrac{\delta(3\alpha-\beta)}{16\alpha} \\
R_{II} &= \tfrac{\delta^3}{16\alpha\beta} \\
R_{III} &= \tfrac{\delta(3\beta-\alpha)}{16\beta}
\end{aligned}
$$

Since we require in this case that all lobby groups are active we need $R_I \geq 0$ and $R_{III} \geq 0$, which is ensured with $3\alpha \geq \beta \geq \frac{\alpha}{3}$. Since the preference of players 1 and 4 coincide with ρ_I and ρ_{III} respectively, neither of them would prefer to free ride.

Next, we examine players 2 and 3 who have agreed on some $\rho_{II} \in [\rho_2, \rho_3]$. If one player abandoned the coalition, the other player would not continue to lobby be herself due to Proposition 1.1, so the altered lobby group structure would only contain players 1 and 4 each being a group by themselves. In this case the expected utility for both players 2 and 3 would be $1/2d_{iI} + 1/2d_{iII} = \frac{2-\delta}{2}$. In comparison, we calculated the expected utility of coalition formation for players 2 and 3 with $R_I + R_{II} + R_{III} = \delta/2$:

$$\begin{aligned}
u_2(\sigma, \mathcal{R}) &= \tfrac{4[(1-d_{2I})(3\alpha\beta-\beta^2)+(1-d_{2II})\delta^2+(1-d_{2III})(3\alpha\beta-\alpha^2)]-\delta^3}{32\alpha\beta} \\
u_3(\sigma, \mathcal{R}) &= \tfrac{4[(1-d_{3I})(3\alpha\beta-\beta^2)+(1-d_{3II})\delta^2+(1-d_{3III})(3\alpha\beta-\alpha^2)]-\delta^3}{32\alpha\beta}
\end{aligned}$$

Exchanging $d_{2I} = \alpha - d_{2II}$, $d_{2III} = \beta + d_{2II}$, $d_{3I} = d_{3II} + \alpha$, and $d_{3III} = \beta - d_{3II}$ leads to

$$u_2(\sigma, \mathcal{R}) = \tfrac{4[d_{2II}(\alpha^2-\beta^2-\delta^2)+8\alpha\beta-2\alpha\beta\delta]-\delta^3}{32\alpha\beta} \tag{10}$$

$$u_3(\sigma, \mathcal{R}) = \tfrac{4[d_{3II}(\beta^2-\alpha^2-\delta^2)+8\alpha\beta-2\alpha\beta\delta]-\delta^3}{32\alpha\beta} \tag{11}$$

This expected payoff is at least as good as free riding, if the condition *b*) is fulfilled for player 2 and *c*) for player 3 respectively. □

The interpretation of this proposition is a measurement of the minimal distance between active lobby groups, before free riding starts to dominate. In particular the first condition states that the corner lobby groups would drop out, if the center coalition would approach their position too closely. Note that in contrast to the previous constellations the rather moderate lobby group with two members can crowd out the rather extreme lobby group which only has one member.

Condition *b*) and *c*) of proposition (2) consider the players who form a coalition. After the lobby group formation players 2 and 3 have to find a mechanism to agree upon the lobby group's political preferences ρ_{II}. Both conditions state that distance towards each other has to be smaller than relative distance to the corner lobby groups. In other words, these conditions answer the question what the maximal concession of players 2 and 3 can be for the negotiation about ρ_{II}. Indeed, we can be more precise on the willingness of these two players to pull their resources together.

Proposition 3. *Player 2 and 3 form a coalition, iff opting for ρ_2 still dominates free riding for player 3 and vice versa.*

Proof. In order to compare equations (10) and (11) we need to adjust the alphas and betas of the players, since they compare two cases, when the lobby group either adopts player 2's or player 3's initial preferences. Concretely, we exchange player 2's α with $\alpha + d_{2II}$ and player 3' β with $\beta + d_{3II}$. We get:

$$d_{2II} = \frac{1/4\delta^3 - 2(\alpha + d_{2II})^2\beta - 4(\alpha + d_{2II})\beta^2 - 2\beta^3}{(\alpha + d_{2II})^2 - \beta^2 - \gamma^2} - \beta \text{ and}$$
$$d_{3II} = \frac{1/4\delta^3 - 4\alpha^2\beta + d_{3II}) - 2\alpha(\beta + d_{3II})^2 - 2\alpha^3}{(\beta + d_{3II})^2 - \alpha^2 - \gamma^2} - \alpha$$

Both equations are fulfilled with equality if $d_{2II} = d_{3II}$ $\square$

The logic of the coalition formation can be described as follows: We can neglect the outcome of the negotiations between players 2 and 3 about ρ_{II}, because for both of them the other players position is the tipping point between coalition formation and free riding. Loosely speaking, a player could formulate her strategy of finding potential collaborators in such a way: If another player is not willing to support me with my claim, then she is not worth to form a coalition with. However, if that player would support my claim, then any agreement on ρ_{II} is better than free riding.

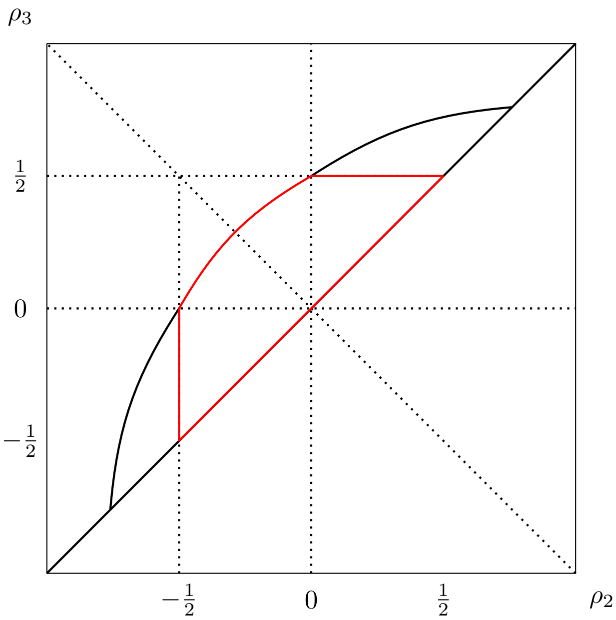

Figure 2: Stable lobby group structures of Proposition (2)

Figure 2 illustrates all possible allocations of preferences within the red lines that can induce a stable lobby group structure as it is described by Proposition (2). For this figure and the ones to come we fix $\rho_1 = -1$ and $\rho_4 = 1$ and vary for ρ_2 and ρ_3. Note that the bottom right part of the figure is blank, since we assume $\rho_2 \leq \rho_3$. Further, this figure would look the same if we also vary ρ_1 and ρ_4, just with different scaling of the axis.

Next, we examine the case when there is a coalition of players at the corner, i.e. either players 1 and 2, or 3 and 4 form a coalition. For symmetric reasons we only examine player 1 and 2 forming a coalition. Then the expected payoff function of the lobby groups are:

$$\begin{aligned}
\pi_I(\sigma,\mathcal{R}) &= \frac{R_I + (1-\alpha)R_{II} + (1-\delta)R_{III}}{R_I + R_{II} + R_{III}} - 1/2R_I \\
\pi_{II}(\sigma,\mathcal{R}) &= \frac{(1-\alpha)R_I + R_{II} + (1-\beta)R_{III}}{R_I + R_{II} + R_{III}} - R_{II} \\
\pi_{III}(\sigma,\mathcal{R}) &= \frac{(1-\delta)R_I + (1-\beta)R_{II} + R_{III}}{R_I + R_{II} + R_{III}} - R_{III}
\end{aligned}$$

Following the same pattern, we get this result:

Proposition 4. *A lobby group structure $\sigma = \{S_I, S_{II}, S_{III}\}$ with either $z_I = 2$ or $z_{III} = 2$ is not stable.*

Proof. The FOCs of the lobby groups read:

$$2\alpha R_{II} + 2\delta R_{III} = (R_I + R_{II} + R_{III})^2 \tag{12}$$

$$\alpha R_I + \beta R_{III} = (R_I + R_{II} + R_{III})^2 \tag{13}$$

$$\delta R_I + \beta R_{II} = (R_I + R_{II} + R_{III})^2 \tag{14}$$

Again, we solve this system of equations with:

$$\begin{aligned}
(12)\&(14) \;\Rightarrow&\quad R_I = 2R_{II} + (2 + \beta/\alpha)R_{III} \\
(13)\&(14) \;\Rightarrow&\quad R_I = R_{II} - R_{III}.
\end{aligned}$$

However, these results yield $3R_{II} = (-1 - \beta/\alpha)R_{III}$, such that we can only solve the system of equations with $R_I = R_{II} = R_{III} = 0$ which contradicts the assumption of active lobby groups. □

We can now formulate a more general insight about the center players.

Corollary 1. *Free riding is the dominant strategy for a player who is by herself between two lobby groups.*

Proof. Here, we simply add player 2's and 3's strategy of Proposition (1) and (4). □

iv) *Two lobby groups and one coalition:*

In this constellation there are two lobby groups, while one group consists of one member and the other is coalition of either two or three players. Without loss of generality assume that player 1 is the single member of lobby group I. Then we get these two expected profit functions:

$$\pi_I(\sigma, \mathcal{R}) = \frac{R_I + (1-\alpha)R_{II}}{R_I + R_{II}} - R_I$$
$$\pi_{II}(\sigma, \mathcal{R})\frac{(1-\alpha)R_I + R_{II}}{R_I + R_{II}} - \frac{1}{s_{II}}R_{II}$$

This results in the optimal investment of the lobby groups

$$R_I = \quad z_{II}\alpha/(z_{II}+1)^2$$
$$R_{II} = \quad \alpha/(\tfrac{1}{z_{II}}+1)^2$$

Before we turn to the stability of this lobby group structure we add another constellation.

v) *Two lobby groups and two coalition:*

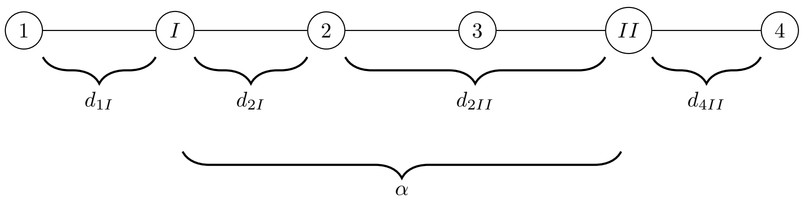

Figure 3: Two lobby groups and two coalitions

Here both players 1 and 2 as well as players 3 and 4 form a coalition each. There is no free riding. The optimal investment of the lobby groups are $R_I = R_{II} = \frac{\alpha}{2}$. For both constellations we can formulate this result:

Proposition 5. *In a lobby group structure $\sigma = \{S_I, S_{II}\}$ there can only be coalition formation if $z_I = z_{II} = 2$ and*

- $3d_{2I} \geq d_{2II}$
- $4d_{1I} \geq 5d_{2I} + d_{2II}$
- $4d_{4II} \geq d_{3I} + 5d_{3II}$
- $3d_{3II} \geq d_{3I}$.

Proof. First we show that there cannot be coalition formation in *iv*). It is sufficient to show that for player 2 free riding dominates coalition formation, if S_I contains only player 1. Consider a coalition of players 2 and 3. The expected utility of coalition formation for player 2 with $\alpha = d_{2I} + d_{2II}$ is $u_2 = (9 - 5d_{2I} - 8d_{22})/9$. Player 3 would still be active and adopt $\rho_3 = \rho_{II}$, if player 2 abandoned S_{II}, so with $\alpha = d_{3I}$ free riding yields an expected utility of $(2 - d_{3I})/2$. Using $d_{3I} = d_{2I} + d_{2II} + d_{3II}$ player 3 compares expected utility of coalition formation $u_3 = (9 - 8d_{2I} - 8d_{2II} - 9d_{3II})/9$ with expected utility of free riding $u_3 = (2 - d_{2I})/2$, since player 2 would also continue to lobby by herself. These two inequalities ensure that coalition formation dominates free riding for player 2 and player 3 respectively:

$$d_{3II} \geq 1/9 d_{2I} + 7/9 d_{2II}$$
$$d_{3II} \leq 1/9 d_{2I} - 8/9 d_{2II}$$

Both inequalities cannot be satisfied, so regardless how players 2 and 3 agree on some ρ_{II}, one of them will always prefer to free ride. This logic obviously also holds true if we consider a coalition of players 2, 3 and 4.

Next, we examine constellation v). Replacing $\alpha = d_{2I} + d_{2II}$ in the optimal investment for players 1 and 2 and $\alpha = d_{3I} + d_{3II}$ for players 3 and 4 gives us these expected utilities for coalitions formation.

$$\begin{aligned} u_1 &= (4 - d_{1I} - 3d_{1II})/4 \\ u_2 &= (4 - 3d_{2I} - 3d_{2II})/4 \\ u_3 &= (4 - 3d_{3I} - 3d_{3II})/4 \\ u_4 &= (4 - 3d_{4I} - d_{4II})/4 \end{aligned}$$

If instead player i chooses to leave the coalition with player j, it would be worthwhile for player j member to continue by herself and invest in lobbying for ρ_j. Expected utility for player i then becomes $1/3(1 - \rho_j) + 2/3(1 - \rho_k)$, $j \notin S_k$, such that

$$\begin{aligned} u_1 &= (3 - d_{1I} - 2d_{1II} - d_{2I})/3 \\ u_2 &= (3 - d_{1II} - 3d_{2II})/3 \\ u_3 &= (3 - d_{3I} - d_{4I})/3 \\ u_4 &= (3 - d_{3II} - 2d_{4I} - d_{4II})/3 \end{aligned}$$

Comparing the utility levels for each player gives the conditions listed in (5). □

The intuition of this proposition has some traits of a prisoner's dilemma. If players consider to form a coalition against a lobby group with only member, then there will always be free riders. If instead that coalition for some reason consists of two members, then the both remaining players may both benefit from working together. However, in contrast to constellation iii) the two cooperating players actually need a certain degree of disagreement about their preferred policy. Concretely, the four conditions of (5) list the minimal amount of concession that each players 1 to 4 respectively have to do. We can interpret this with the following example: Suppose players 1 and 2 have rather similar preferences and face a lobby group that consists of players 3 and 4, which lobbies for an opposing policy. Then both players 1 and 2 would be willing to invest, but at the same time the other player would choose to free ride. Next, also suppose that player 3 and 4 joined together for an opposing policy, but players 1 and 2 do not have similar preferences. Then both players are willing to make a compromise concerning ρ_I in exchange for sharing the expenses. Interestingly, the lobby group structure with two coalitions would also be stable, if players 2 and 3 have identical preferences as long as the distances towards players 1 and 4 are sufficiently large.

The more general insight is that players are favoring coalition formation, if there already exist opposing lobby groups with greater financial capacities.

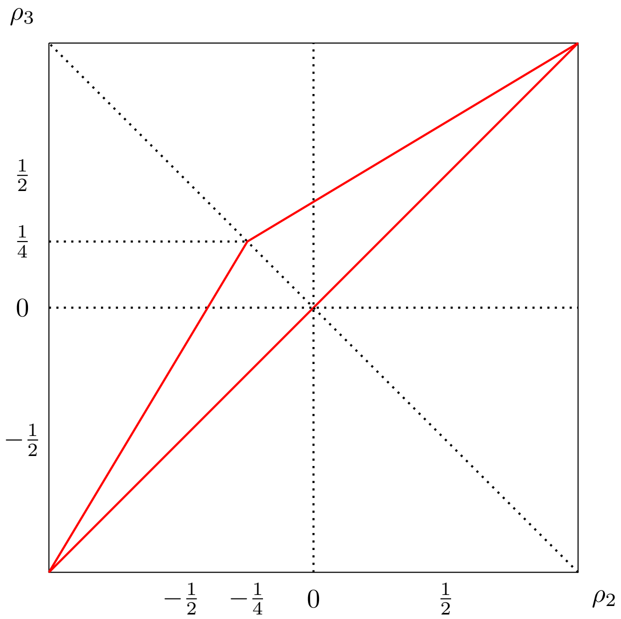

Figure 4: Stable lobby group structures of Proposition (5)

Figure 4 illustrates within the red lines all possible allocation of preferences that induce a stable coalition structure of Proposition (5).

vi) *Two lobby groups with one member each:*

There are two lobby groups active and both of them have only one member, while two the remaining players are free riders. There are six different possibilities how the active players and the free riders can be allocated. There can be either both center or both corner player be active. Then it could be the neighboring players at the corners on both sides, or one corner and one center player. If player i is active and player j is a free rider the expected utilities read $u_i = (4 - 3\alpha)/4$ and $u_j = (2 - d_{jI} - d_{jII})/2$. This leads to these stable lobby group structures with d_{23} denoting the distance between players 2 and 3:

Proposition 6. *A lobby group structure $\sigma = \{S_I, S_{II}\}$ with $z_I = z_{II} = 1$ is stable with active players*

1 and 4 *if* $d_{23} \geq \frac{1/4\alpha^3 - 2\alpha d_{3I} d_{3II}}{d_{3I}^2 - d_{3II}^2 - \alpha^2}$ *or* $d_{23} \geq \frac{1/4\alpha^3 - 2\alpha d_{2I} d_{2II}}{d_{2I}^2 - d_{2II}^2 - \alpha^2}$

2 and 3 *if* $d_{1I} \leq d_{1II}/2$ *and* $d_{4II} \leq d_{4I}/2$*,*

1 and 3 *if* $d_{4II} \leq d_{4I}/2$*,*

2 and 4 *if* $d_{1I} \leq d_{1II}/2$*,*

1 and 2 *if* $9d_{3II} \leq d_{3I}$*, and* $d_{4II} \leq d_{4I}$ *and*

3 and 4 *if* $9d_{2II} \leq d_{2III}$ *and* $d_{1I} \leq d_{1II}$*.*

Proof. If the corner players form lobby groups, Corollary 1 ensures that the center players will free ride by themselves. They will not form a coalition either, if either condition b) or c) of (3) is not satisfied. Whenever there is a corner player by herself next to a lobby group, the only comparison is to either free ride or to replace it. Then free riding is dominant if the distance towards the rather favoring lobby group is smaller than towards the opposing one. If the neighboring players at the corner have formed a lobby group, the same holds true of the remaining players by themselves. In addition, coalition formation among these needs to be is prevented for the lobby group structure to be stable. Then it is sufficient to ensure that the remaining center player prefers to free ride. Let players 1 and 2 form coalition I and II, then expected utility of coalition formation for player 3 yields $u_3 = (9 - 5d_{3I} - 8d_{3III})/9$. Noticing that S_{II} would be replaced by S_{III} player 3 would prefer to free ride if , $9d_{3II} \leq d_{3I}$ even in player 4 would agree to vote for ρ_3. With symmetry the same argument holds for player 2, if players 3 and 4 form lobby groups. □

Generally, we note that free riding takes place every time when players have similar preferences. We notice that the same distribution of political preferences can lead to different outcomes. If two players share a political view then it would be a stable lobby group structure if either of them lobbies by herself while the other one free rides.

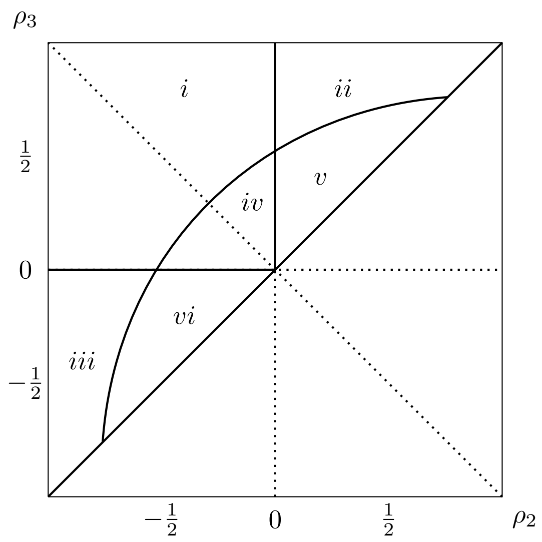

Figure 5: Stable lobby group structures of Proposition (6) part I

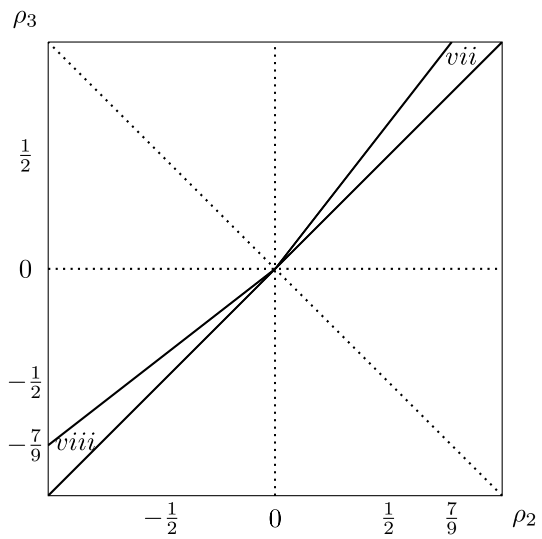

Figure 6: Stable lobby group structures of Proposition (6) part II

In figure 5 we summarize various possibilities how stable lobby group structures could arise following Proposition (6). These are:

Area	Players that are lobbyists in a stable structure
i	1 and 3, 1 and 4, 2 and 3, or 2 and 4
ii	1 and 3, or 1 and 4
iii	1 and 4, or 2 and 4
iv	1 and 3, 2 and 3, or 2 and 4
v	1 and 3
vi	2 and 4

In addition we find in figure 6 stable lobby groups structure with players 1 and 2 lobbying in area vii and players 3 and 4 in area $viii$.

vii) *One lobby group or no lobby:*

Just for completion we include the case when there is only one or even no lobby group. By nature of a lottery, there will always be at least one player active. Further, one lobby group can only occur if all players have identical preferences. As soon as there is already a little disagreement between i and j and i becomes active, the expected profit function of player j yields $\alpha/2 > 0$. However, the lottery would make little sense in either scenario, since an arbitrary low amount of resources would be sufficient to win it.[6].

5 Conclusion

We have constructed two stage rent-seeking contest where four players can form lobby groups in the first stage and all formed groups enter the contest in the second stage. The players can either choose to lobby be themselves, form a coalition or free ride. Regardless of their activity, the outcome of the contest affects all players. We then analyzed all possible constellation that could arise when the players decide. A lobby group structure is called stable, if no player is better of deviating by herself and no two players want to form a coalition either.

Based on the preferences on the players, we find that various constellations can be stable. The reason for this is that players find it worthwhile to free ride if there is already a lobby group close to her own preferences. Coalition formation can only occur in two setting: On the one hand the first two and the last two players can form a group each in order to have a balanced power in terms of resources. On the other hand the players with rather moderate preferences can form a coalition of both players with rather extreme preferences are active. It is also the only constellation that contains three lobby groups.

This insight can be interpreted beyond our model. Whenever multiple lobby groups are active, then those at the political edges require fewer members than the middle ones. In addition, the distance between any of the groups needs to surpass a certain threshold, otherwise some groups or

[6] Here we allow for some mathematical sloppiness.

group members will rather free ride.

With a clearer understanding of the interplay between free riding and participation in lobbying activities, this contribution includes some implications. There is a tendency that players with rather extreme preferences get more involved. However, given sufficient resources of the players the group formation induces a certain counterbalance by itself. Hence, the regulation of lobbying should be limited to ensuring equal access to the groups. In addition, an implementation of rather extreme positions can be prevented if the corner players both are part of coalition. Therefore the inclusion of more players helps to establish a mediating policy.

For future research we could compute a similar approach but with an asymmetric contest, such that some players or groups have different valuations, costs or generate a greater impact with their resources. Further, it would be interesting to apply the results of this paper to the discussion about the effects of lobbying on the legislation process and/or on social welfare.

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